An Operational Space Tracking Algorithm for Generating Dynamic Simulations of Movement

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1. ABSTRACT

The objective of the current project was to develop a computationally practical method for generating forward dynamic simulations of three-dimensional movements that track experimental data while allowing for bias errors in joint angle measurements. An operational space control framework was adapted from robotics and used to drive a 20 degree-of-freedom, torque-actuated simulation of normal gait. Joint torques were computed that precisely tracked the trajectories of the feet (end-effectors) while also generally tracking the angular displacements of the lower-extremity joints. The algorithm generated a forward dynamic simulation of gait using about 30 minutes of computer time. During stance, adjustments to the experimental joint angles were introduced that removed penetration of the feet into the ground and sliding of the feet relative to the ground. Simulated joint torques were similar in both shape and magnitude to what has been computed using inverse dynamics. We believe that an end-effector control formulation provides a powerful simulation framework in biomechanics in which movement can be described with greater emphasis on the task rather than on traditional joint-space representations.

2. INTRODUCTION

Biomechanical simulations of movement typically use muscle or torque actuators to drive a linked segment representation of the body. A major challenge in using dynamic simulations is finding a set of actuator controls that produce purposeful movement. One such problem involves deriving a set of controls that can track body motions and external reaction forces measured experimentally (Neptune and Hull 1998, Kaplan and Heegard 2001). Some of the difficulties encountered in this tracking problem arise from experimental limitations. In particular, skeletal and joint motion cannot be measured directly but must be inferred from the motion of markers attached to the skin. Inaccuracies result from soft tissue movement, equipment noise, and imprecise knowledge of underlying joint mechanics. Such inaccuracies are particularly problematic when using inverse dynamics techniques since small offsets in joint locations can cause large errors in estimated joint torques (Holden and Stanhope, 1998). Consequently with inverse dynamics techniques, there is no guarantee that estimated joint torques are accurate or would replicate measured movement when input to a forward dynamics model (Risher et al. 1997).

Thus there is a need for new tracking algorithms in biomechanics that can accommodate motion measurement errors and can also integrate inverse and forward dynamic techniques to ensure self-consistency between estimated actuator forces and limb motions. The current study was undertaken to begin to address these issues. The

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starting point for the study is the recognition that external force measurements provide some of the most reliable measures in a biomechanics lab, and further that the trajectory of the feet should be consistent with these force measurements. Operational space control, adapted from robotics (Khatib 1987), was used to produce a torque-actuated forward dynamic simulation of gait that produced desired trajectories of the feet while generally tracking the angular displacements of the joints in the lower extremity.

3. METHODS

3.1 Experimental Data

Joint angles and ground reaction forces obtained from five healthy young adult males (Anderson and Pandy, 2001) were averaged and used as inputs to the tracking algorithm. Video and force-plate data were recorded simultaneously. Kinematic data were recorded using a four-camera, passive video system (Motion Analysis Inc., Santa Rosa, California). Ground-reaction forces and moments were sampled at 1000 Hz using a six-component, strain-gauge force plate (Bertec, Inc., Columbus, Ohio).

3.2 Forward Dynamics Model

A three-dimensional model of the human body with 20 independent degrees of freedom (dof) was used in this study (Anderson and Pandy 2001). Six generalized coordinates were used to describe the position and orientation of the pelvis relative to a ground reference frame. Eight lower-extremity segments branched out in an open chain from the pelvis. The hips were modeled as 3 degree of freedom (dof) ball-and-socket joints. A planar knee joint allowed for sliding-rolling between the femur, tibia and patella. Pin joints about anatomically oriented axes represented the ankle, subtalar and metatarsal joints (Delp et al. 1990). Equations of motion for the linked segment model were derived using Dynamics Pipeline (MusculoGraphics, Inc., Chicago, IL) and SD/FAST (Symbolic Dynamics, Inc., Mountain View, CA).

The dynamical equations of motion for the 20 dof model may be expressed as:

$$A(q)\ddot{q} + b(q,\dot{q}) + g(q) = \tau \,, \tag{1}$$

where q, $\dot{q}$ and $\ddot{q}$ are the joint angular displacements, velocities and accelerations; $A(q)$ is the mass matrix; $b(q,\dot{q})$ is a vector of Coriolis and centrifugal forces; $g(q)$ is a vector of gravitational forces; and τ is a vector of generalized forces associated with each degree of freedom.

3.3 Optimal Foot Trajectories

The starting point in the tracking algorithm was generating a set of optimal foot trajectories that when tracked during a simulation would yield the experimentally recorded ground reaction forces. Nonlinear constrained optimization was used to accomplish this task. Foot-floor contact was modeled with 16 nonlinear viscoelastic springs (Gerritsen and van den Bogert 1995) distributed across the bottoms of the feet. Shear forces in the ground plane were modeled by linear spring-dampers with moving set points (Anderson and Pandy 2001). Each foot was given six degrees of freedom to describe the translation and orientation of the foot with respect to ground ($X = [x\; y\; z\; \theta_1\; \theta_2\; \theta_3]^T$). The optimization problem was to find values of X that generated the experimental ground reaction force and that minimized the difference between the model and experimental foot coordinates. The performance criterion J was chosen to be the weighted sum of the squared differences in foot coordinates:

$$J = \sum_{i=1}^{6} w_i (X_i^{opt} - X_i^{\exp})^2 , \tag{2}$$

where X_i^{opt} and $X_i^{\exp}$ are the optimal and experimental foot coordinates, respectively.

3.4 Tracking Formulation

Operational space control (Khatib 1987) was used to track the optimal foot trajectories. In operational space control, equations of motion are derived for the end-effector and used as the basis for generating joint torques that are required to track desired end-effector kinematics (Khatib 1987). In the current application, the feet were treated as the end-effectors. The equations of motion for the feet can be expressed as:

$$\Lambda(X)\ddot{X} + \mu(X,\dot{X}) + p(X) = F_e , \tag{3}$$

where $\Lambda(X)$ is the mass matrix of the end-effector, $\mu(X,\dot{X})$ is the vector of end-effector centrifugal and Coriolis forces, $p(X)$ is a vector of gravity forces, and F_e is the net generalized force vector acting on the end-effector. At any given configuration, the end-effector mass matrix can be computed from the mass matrix of the system

$$\Lambda^{-1}(q) = J(q)A^{-1}(q)J^T(q) , \tag{4}$$

where J is the jacobian relating the velocities of the end-effector to the time derivatives of the generalized coordinates:

$$\dot{X} = J(q)\dot{q} . \tag{5}$$

The control of the feet in operational space was achieved by designing a set of generalized operational forces F_e to track the optimal foot trajectories and projecting them onto required joint torques using

$$\tau_e = J^T(q)F_e . \tag{6}$$

The desired acceleration in each degree of freedom was computed using the optimal foot accelerations with linear corrections for errors in the foot positions and velocities:

$$\ddot{X}^* = \ddot{X}^{opt} + k_p(X^{opt} - X) + k_v(\dot{X}^{opt} - \dot{X}) , \tag{7}$$

where k_p and k_v are feedback gains that determined how aggressively position and velocity errors were corrected. These gains were selected to produce a critically damped second order responses in the tracking errors (i.e., $k_v = 2\sqrt{k_p}$). The contact forces acting on the feet, F_c, were taken directly from the experimental ground reaction and included in an end-effector control law:

$$F_e = \Lambda(x)\ddot{X}^* + F_c . \tag{8}$$

The net joint torques required to track the foot trajectories were then computed by also adding on those torques necessary to overcome gravity, Coriolis, and centrifugal forces:

$$\tau_f = J^T(q)F_e + b(q,\dot{q}) + g(q)\,. \tag{9}$$

Since the number of foot coordinates (12) is less than the number of degrees of freedom in the model (20), the system is still redundant even when employing foot tracking. That is, the whole body kinematics required to track the optimal foot trajectories is not fully determined. Therefore an additional set of null-space joint torques can be added to the operational space control torques that do not affect the foot tracking but that can achieve additional tracking objectives. In our case, we would to track the measured joint angles. Therefore, a set of joint torques were computed based on the current error between the experimental and simulated joint angles and angular velocities:

$$\tau_j = k_p(q^{\exp} - q) + k_v(\dot{q}^{\exp} - \dot{q})\,. \tag{10}$$

The joint angle tracking torques were then projected into the null space of the operational space force vector such that they do not affect the optimal foot tracking (Khatib, 1987):

$$\tau_j^{ns} = [I - J^T(q)\bar{J}^T(q)]\ \tau_j\,. \tag{11}$$

$\bar{J}$ is the generalized inverse of the Jacobian matrix that minimizes the manipulator's instantaneous kinetic energy and can be computed from the Jacobian and mass matrices:

$$\bar{J}(q) = A^{-1}(q)J^T(q)\Lambda(q)\,. \tag{12}$$

The net generalized forces applied to the model within the forward dynamic simulation were then given by:

$$\tau = \tau_e + \tau_j^{ns}\,. \tag{13}$$

4. RESULTS

The tracking algorithm produced a forward dynamic simulation that reproduced the salient features of gait. By using operational space control, corrections to the experimental joint angles were introduced that removed penetration of the feet into the ground and sliding of the feet relative to the ground during stance (Figure 1). During much of stance, differences between the measured and simulated joint angles were less than 10 degrees; however, prior to toe-off when the heel was rising, the needed corrections were larger and took place primarily at the ankle (Figure 2). These adjustments to ankle angle persisted throughout much of swing, but had little impact on the joint torques. The joint torques estimated using the tracking algorithm were similar in magnitude and shape to those commonly obtained by inverse dynamics (Figure 3).

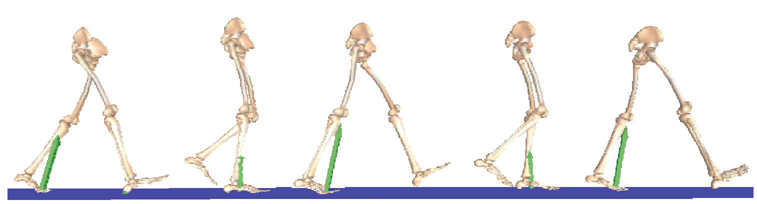

Fig 1. Gait pattern of the forward dynamic simulation. End-effector motion control was used to ensure that the feet did not penetrate or slide relative to the ground during stance.

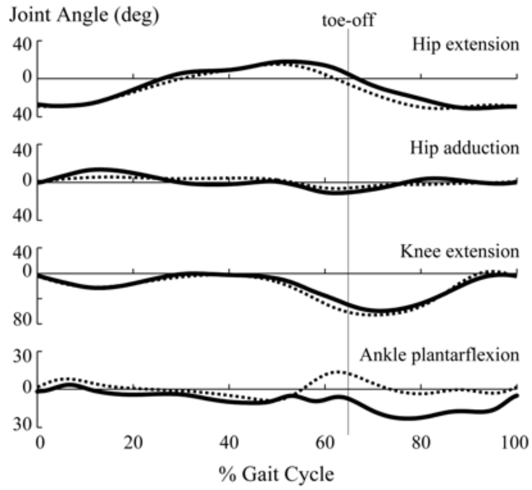

Fig. 2. Simulated (black) and experimental (dashed) joint angles. Simulated angles were within 10 degrees during most of stance. Just prior to toe-off, large corrections were introduced at the ankle.

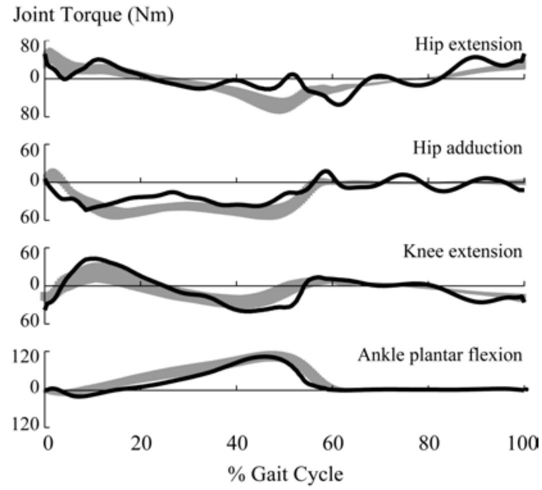

Fig. 3. Joint torques determined by the tracking algorithm (black lines) and computed by inverse dynamics techniques (Judge et al. 1996).

5. DISCUSSION

The tracking algorithm employed in this study provides a computationally practical method for generating realistic forward dynamic simulations of complex three-dimensional movement from experimental data. The simulation that was obtained for gait required about 30 minutes of computer time. The key to obtaining this performance is the use of on-going feedback during the simulation; only one simulation pass through the movement is needed. Open-loop tracking methods that require iteration can take substantially more computation time (Neptune and Hull 1998).

A strength of the approach is the use of operational space control. Given knowledge of constraints that the end-effectors must satisfy, operational space control provides a method for correcting bias errors in measured kinematics based on the dynamic properties of the system. In gait, for example, it is known that the feet do not penetrate the ground. In operational space, the motion of the feet can be controlled very accurately to enforce this constraint. Further, by projecting into the null space of the operational space force vector, the ankle, knee, and hip joints can be controlled without adversely affecting the motion of the feet. An operational space framework also offers the possibility of reducing the reliance on joint-space representations of movement in favor of descriptions that are more closely tied to the task. An end-effector control representation can be more accurate, convenient, and is perhaps more reflective of how the nervous system performs motor tasks (Hogan et al. 1987).

The tracking algorithm does possess limitations. In particular, the algorithm is sensitive to delays in actuator response. Physiological delays in force production due to muscle activation and contraction dynamics are on the order of 20-100 milliseconds (Zajac, 1989). In the simulation performed in this study, such delays were sufficient to cause significant errors between simulated and desired kinematics. Therefore, in order to obtain simulations that are actuated by muscles with physiologically realistic force response latencies, the tracking algorithm would need to incorporate some anticipation of the actuator outputs demanded. Additionally, the algorithm cannot be used in its current form to generate self-balancing simulations of whole-body movement. During the simulation, a set of residual generalized forces were applied directly to the pelvis in order to track all of the degrees of freedom of the model. While a portion of these residual forces represents the un-modeled dynamics of the upper body, a portion likely was also needed to correct for numerical errors and inconsistencies between the model and the subjects.

Despite its current limitations, we believe the tracking approach presented in this work has many potential applications within the field of biomechanics. The incorporation of forward and inverse dynamics within a single approach offers a computationally practical way of obtaining forward dynamics simulation and ensures that inverse dynamics results are consistent with model assumptions. Once a forward dynamics solution has been obtained, one can then perturb the controls to ascertain their influence on coordination and also use the solution as a starting point for more traditional dynamic optimization approaches. Finally, by using feedback and exploiting end-effector control, one has a framework for the theoretical study of human movement that more closely reflects how people really move.

6. ACKNOWLEDGEMENTS

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