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Short communication

Limitations in the use and interpretation of continuous relative phase

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Abstract

Continuous relative phase (CRP), a variable used to quantify intersegmental coordination, is difficult to interpret if care is not taken regarding the assumptions and limitations of the measure. Specifically, CRP is often interpreted as a higher resolution form of discrete relative phase (DRP). DRP, however, yields information regarding the relative dispersion of events in oscillatory signals while CRP describes their relationship in a higher order phase-plane domain. In this paper we address issues surrounding the calculation of CRP and suggest a new interpretation based on the aforementioned methodological issues. Through the use of test signals, with known properties, it was found that the CRP information will be arbitrary if no normalization procedures are used to account for frequency differences in the component oscillators. In addition, signals with non-sinusoidal trajectories will produce patterns in CRP that are not equivalent to discrete relative phase (DRP) measures. The implications of these issues are discussed.

Keywords: Coordination; Continuous relative phase

1. Introduction

Traditionally, measures of relative phase have been used to quantify the coordination between two or more segments during an activity (Von Holst, 1973). Von Holst examined the coupling between fins in fish and quantified phase as the temporal difference between successive inflection points in the oscillations of the fins. This measure is today typically referred to as discrete relative phase (DRP). Another method used to quantify phase relationships is continuous relative phase (CRP) (Kelso, 1995). CRP is typically derived from the position—velocity phase-planes of two predominantly sinusoidal oscillators. Since Kelso's (1995) reports on the dynamics of finger oscillations, many other types of behavior have been characterized by the use of CRP. These include such activities as swinging wrist pendula (Amazeen et al., 1998), juggling (Post et al., 2000) and trunk and pelvis coordination during walking (Van Emmerik et al., 1999).

One main issue in utilizing CRP is how it is interpreted. Kelso (1995) essentially utilized CRP as a higher resolution form of DRP where the instantaneous phase of two oscillators could be determined across multiple points of a cycle. This is not the same as DRP where an event is chosen over which relative phase is examined (i.e. peak or inflection point). Kelso showed that in finger oscillations CRP and DRP were almost identical.

This congruence between CRP and DRP disappears as oscillations deviate from being sinusoidal or even in sine waves whose frequencies are other than of $0.5/\pi$ Hz. Thus, CRP will not be equivalent to the relative temporal positions between the waves. This lack of congruence has been referred to as an artifact, where DRP and not CRP is considered the "intuitive result" (Fuchs et al., 1996). To account for this, researchers often normalize the phase-planes prior to the calculation of CRP (Hamill et al., 2000).

As CRP expands from its traditional use with predominantly sinusoidal signals into non-sinusoidal

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activities and partial oscillations (Tomioka et al., 2001; Kurz and Stergiou, 2002), the necessity for phase-plane normalization should be clarified and the limitations on the interpretation of CRP information should be reviewed. Thus, the purpose of this paper is twofold: (1) to illustrate the need for phase-plane normalization prior to calculating phase angles in the waveforms; and (2) to show that the interpretation of CRP information should be limited to describing the relationship between the individual phase-planes of the two signals and not be used to describe a relationship in their original time series data.

2. Methods

CRP was calculated from test signals with known phase and frequency properties. The use of test signals allowed for a comparison between the calculated CRP values and the intuitive outcomes based on DRP measures.

CRP was calculated as the difference in the phase angles of the test signals. The phase angles used in this exercise were determined from the signals' position vs. velocity (x vs. x') phase-planes. The position (x) time series values were numerically determined at a resolution of 10,000 points per cycle and the velocity (x') values were calculated using a 3-point central-difference technique. From the resulting phase-planes, the phase angle at each point (φ(t)) was calculated relative to the right horizontal using Eq. (1):

$$\varphi(\mathrm{t}) = \tan^{-1}\left(\frac{x'(\mathrm{t})}{x(\mathrm{t})}\right). \quad (1)$$

Three sine waves provided the first set of test signals. The three waves had frequencies of $0.5/\pi$, $1/\pi$ and $2/\pi$ Hz, respectively. All waves had an amplitude of 1.0. $0.5/\pi$ Hz was chosen because the derivative (velocity) of this signal would also have an amplitude of 1.0 and thus produce a circular phase-plane. CRP was calculated for these signals both with and without phase-plane normalization.

For the second set of test signals, CRP was calculated between a skewed sinusoid and the same skewed sinusoid time-shifted by magnitudes of 5% and 35% (18° and 126°) of the period of the original signal.[1] The skewed sinusoid was created using Eq. (2):

$$\text{wave} = \frac{0.41418\cos(\theta - 0.25\pi)}{\sqrt{1 + (0.41418)^2 - (2 \times 0.4148\sin(\theta - 0.25\pi))}} \quad (2)$$

where θ is the time varying state of the oscillator. The resulting wave has a smooth saw-toothed pattern with a $\frac{75}{25}$-duty cycle.

Prior to calculating the phase angles, the phase-planes were normalized by dividing the velocity by $2\pi/p$ where p is the period of oscillation. This results in a circular phase-plane for sinusoidal data and maintains a consistent aspect ratio for all similarly shaped non-sinusoidal waves regardless of their period.

3. Results

This paper was intended to address two main points: (1) normalization, to account for frequency differences between waves, is needed prior to calculating CRP; and (2) CRP between non-sinusoidal signals will not equal DRP.

First, the position—velocity phase-planes of sinusoids at any frequency greater than $0.5/\pi$ Hz will be elliptical along the velocity axis (Fig. 1). When these frequency differences were not normalized, artifacts in the final CRP measure appeared in the form of a low frequency oscillation. Only when normalized did the intuitive result, in terms of DRP terminologies, emerge (Fig. 2).

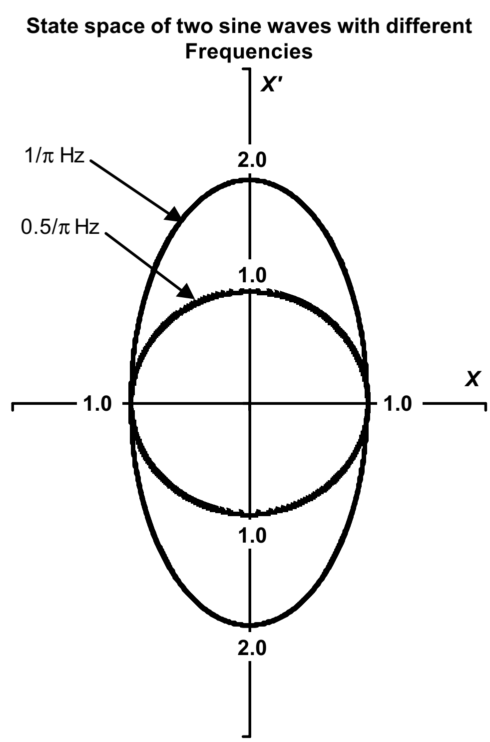

Fig. 1. Trajectories in position-velocity phase-plane of the two sine waves used. The first wave had a frequency of $0.5/\pi$ Hz, with the second sine wave having twice the frequency of the first. After normalizing for the effects of frequency the ellipse (phase-plane of the higher $1/\pi$ Hz frequency wave) is transformed into a circle.

[1] These are equivalent to the DRP relationships as determined in studies by Kelso and Voń Holst.

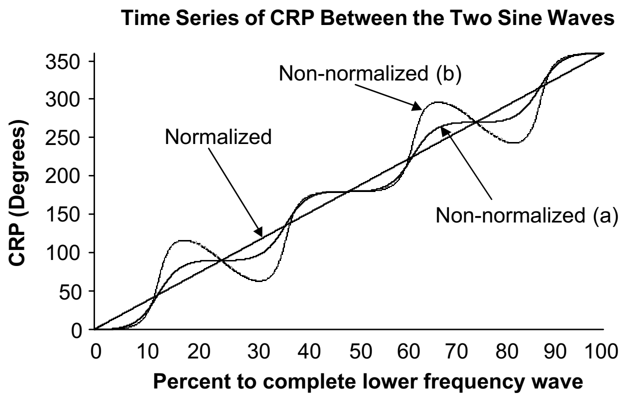

Fig. 2. CRP from the sine waves in Fig. 1 and CRP of the $1/\pi$ vs. $2/\pi$ sine waves. Each is plotted against the percent time it takes the lower frequency wave to complete one cycle both with and without normalization of the higher frequency wave. Because the higher frequency wave in each case completes two cycles in the time it takes the slower frequency wave to complete one, the phase relationship between the two should increase to 360° after a complete cycle of the lower frequency wave. The monotonically increasing phase is observed only after the higher frequency ($1/\pi$ and $2/\pi$ Hz) waves are normalized. With no normalization, artifacts in the known CRP emerge in both the $0.5/\pi$ vs. $1/\pi$ trace (non-normalized (a)) as well as the $1/\pi$ vs. $2/\pi$ trace (non-normalized (b)) in the form of low frequency oscillations. Additionally, the morphologies between the two CRP traces are different with no normalization. This difference is despite the fact that their relative relationships are identical.

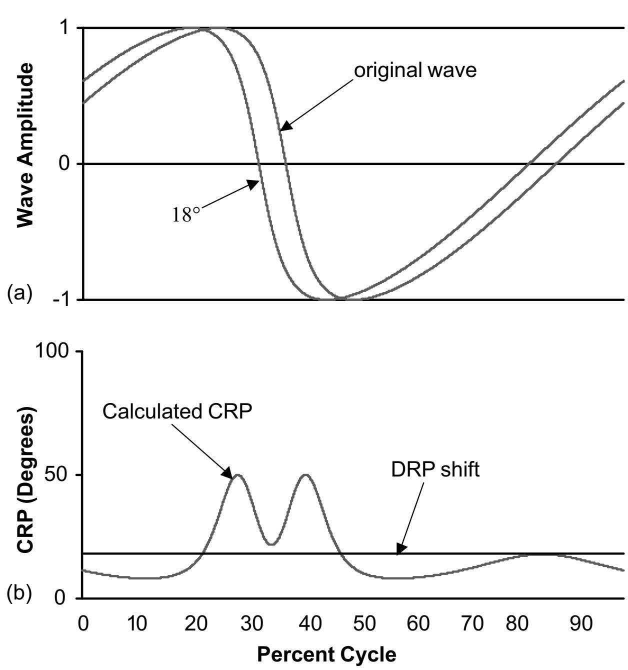

Fig. 3. Time series: (a) of the skewed sinusoid and the same sinusoid phase shifted by 18° and (b) the CRP between the two waves shown in (a). The solid horizontal line in (b) represents the corresponding DRP phase shift between the two component waves.

Additionally, when not normalized, the morphologies between the $0.5/\pi$ vs. $1/\pi$ Hz signals and the $1/\pi$ vs. $2/\pi$ Hz signals were different (Fig. 2).

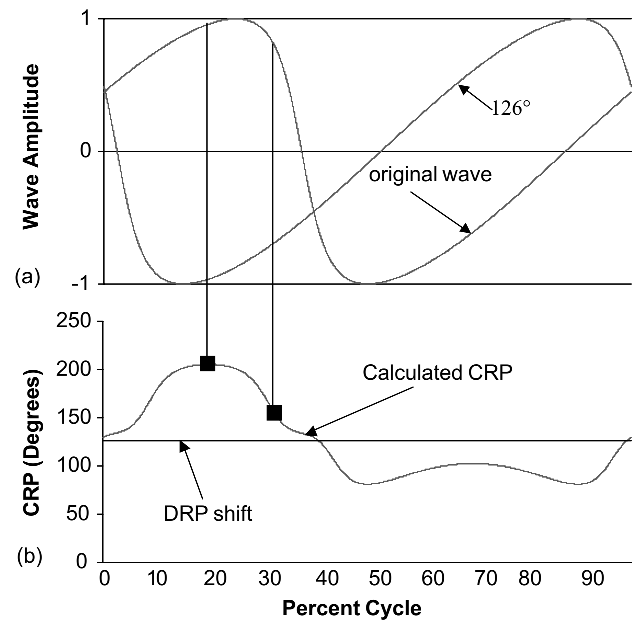

Fig. 4. Time series: (a) of the skewed sinusoid and the same sinusoid phase shifted by 126° and (b) the CRP between the two waves shown in (a). The solid horizontal line in (b) represents the corresponding DRP phase shift between the two component waves. The solid vertical lines represent instances where the signals are moving inphase and antiphase. The inphase and antiphase relationships are not observed in the CRP time series.

Secondly, when non-sinusoidal signals are examined, the interpretation of CRP should be limited to describing the relationship between the individual phase-planes of the signals and not the relationship in the original time-series. A constant time lag between two identical, non-sinusoidal signals did not produce the intuitive DRP relationship when determined via CRP (Figs. 3 and 4). Moreover, the shape of the CRP output is not consistent between the signals with the 18° and 126° phase shifts.

4. Discussion

The results of the CRP calculation show that non-circular phase-planes yield results that are not immediately intuitive. This results from the elliptical (non-circular) nature of the phase planes which are caused by the frequency of the signal (Fuchs et al., 1996). Although CRP will only yield the intuitive result when the phase-planes are circular, we feel consistent results should be emphasized. Even without attempting to achieve a circular phase-plane and the intuitive monotonically increasing phase shift shown in Fig. 2, without some form of normalization, the CRP between the $0.5/\pi$ vs. $1/\pi$ Hz and the $1/\pi$ vs. $2/\pi$ Hz sinusoids will be different (Fig. 2). It is our belief that a coordination measure should be robust enough to yield an identical output when the relationship between the two inputs is identical regardless of the frequency. Consistency will be achieved if the frequency differences between the waves are accounted for via normalization.

Normalization techniques (simple linear scaling of the velocity axis) assume that an oscillatory signal is being used and therefore, may not be appropriate for partial oscillations or non-sinusoidal movements. However, because the frequency components are different between two signals with identical morphologies, normalization to account for these velocity differences should be utilized in these situations as well.

This lack of an intuitive result is caused by the tendency to try to interpret CRP using DRP terminology. DRP provides a comparison of the temporal dispersion of events between two signals while CRP describes their relationship in the phase-plane domain. It is incorrect to assume that they are going to provide the same information and caution should be used when presenting their results. As a result of the different information provided by CRP and DRP, it is incorrect to state that a CRP value near 180° means that the two signals are moving in the opposite directions. This is illustrated in Fig. 4a where, based on the slopes of the time-series data at the vertical indicators, the signals are moving "in-phase" in one case and "out-of-phase" in the next. Although these points indicate very different behaviors in time-series data, they are equidistant from 180° in the CRP output presented in Fig. 4b.

The exact normalization technique used depends on the research question of interest. If the data are sinusoidal, the specific normalization technique is irrelevant. All will scale velocity in a manner where the final result is a circular phase-plane. When the data are not sinusoidal, many different techniques have been utilized, all with the goal of making the phase-plane more circular. Some of these techniques simply rescale the y-axis (Hamill et al., 2000; Burgess-Limerick et al., 1993) while others employ sophisticated transforms (Rosenblum and Kurths, 1998) or non-linear methodologies (Fuchs et al., 1996). The $2\pi/p$ method (used in this paper) will achieve the consistency advocated above. A procedure for normalizing signals with partial oscillations may include interpolating positional data prior to calculating velocity. This would effectively remove frequency discrepancies prior to the differentiation process.

We have shown that normalization is necessary to reduce the effects of frequency when using CRP to compare two predominantly sinusoidal signals. Additionally we have stressed that traditional CRP measures, in non-sinusoids, provide a relationship between the position-velocity phase-planes of two signals and, therefore, cannot reliably be used to describe the relationship between the two signals in the temporal domain. This issue is resolved in consistency between results is the primary focus.

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